PROJECT SUMMARY

Overview:

A tensor is a multilinear map. A decomposition of a tensor T is an expression of T as a sum of "elementary tensors," which are defined according to the application. The rank of T is the smallest number of elementary tensors that can be used to decompose T. In many applications, the elementary tensors are the product tensors, and the rank of a tensor under this choice is called its tensor rank.

Tensors have a wide range of applications in data science and quantum information theory. In data science, if a tensor represents a dataset, then a tensor rank decomposition can be used to both compress and interpret the data. If the tensor rank decomposition is unique, then there is only one consistent interpretation of the data. For this reason, uniqueness is very useful in data processing [25]. In quantum information theory, the product tensors represent unentangled (pure) quantum states, and the tensor rank captures, to some extent, the amount of entanglement present in a state [7].

The PI's first research objective is to characterize the matroidal structure of sets of product tensors. Since a decomposition of a tensor T into product tensors is completely described by that set of product tensors, a natural approach to understanding tensor decompositions is to determine the matroidal structure of such sets. As partial progress toward this objective, the PI conjectured a "splitting theorem" for sets of product tensors [28,29]. He recently proved this conjecture with Fedor Petrov, and used it generalize Kruskal's theorem, a famous sufficient condition for a tensor rank decomposition to be unique [26,31].

The PI's second research objective is to develop our understanding of quantum entanglement as a resource for information processing. In particular, the PI will continue his investigation into identifying the most useful entangled states for fundamental quantum information processing tasks, constructing and certifying entanglement in linear subspaces, describing the invertible maps that can create higher entanglement, and proving bounds on stabilizer rank [24,30,32].

The PI will pursue the aforementioned objectives at Northeastern University under the supervision of the sponsoring scientist, Harm Derksen. In the course of this work, the PI will use methods from matroid theory, algebraic geometry, geometric invariant theory, and representation theory. The sponsoring scientist is a leading expert on relevant aspects of these fields.

Intellectual Merit:

Matroid theory and tensor decompositions are independently quite well-studied topics. However, despite their natural synergy described above, the PI can count on one hand the number of research works that have used matroid theory to study tensor decompositions. The PI's first objective, to formally study this synergy, could unlock a wealth of knowledge about both disciplines. Indeed, the PI's use of matroid theory with Fedor Petrov to generalize Kruskal's theorem serves as strong evidence for this potential.

In the words of Erwin Schrödinger, "[Entanglement is] the characteristic trait of quantum mechanics... that enforces its entire departure from classical lines of thought" [35]. The PI's second objective, to develop our understanding of quantum entanglement as a resource, will both deepen our understanding of the physical universe and play an important role in the development of quantum computers.

Broader Impacts:

During the project, the PI will undertake several initiatives to counteract the gender and socioeconomic disparity that permeates our STEM community. The PI will participate in the "Bridge to Calculus" program for underserved high-school students, which is collaboratively organized by the Northeastern University Mathematics department and the Boston public school system. The PI will also mentor K-12 girls as part of the "Science Club for Girls" 501(c) non-profit program.

The PI will also pursue opportunities to supervise student research. In particular, the PI intends to mentor undergraduate students as part of Northeastern University's MATH 4020, a research capstone project for juniors and seniors.